SUBSTITUTION RATES BETWEEN TECHNOLOGY, LAND AND LABOUR*

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The least-cost method of increasing food supplies is a major concern of most developing countries. In fact, this problem promises to be one of the most urgent problems of the world over the next several decades. Many basic data are required in order that most efficient paths for increasing food output can be selected. There are numerous resource mixes which can be used to increase food output and it would be useful if the marginal rates of substitution relating to all of these resources were known. For example, food supply can be increased by reclamation of more land, use of more fertilizer on given land or both. Many combinations of these alternatives exist. If we knew the marginal rates of substitution between land and fertilizer, or between other sets of technologies and resources, along with the costs of reclamation and fertilizer production and distribution, we could more nearly specify the optimal rates in increasing food supplies.

We provide some basic data on substitution rates of fertilizer for land in this paper. We consider the results to be of basic nature directed towards increasing knowledge and understanding of these phenomena. Of course, much additional data are necessary before quantitative results of this type can be employed in planning and policy. Even then, however, we believe our data begin to represent a small and useful plug in a general "gap of knowledge" which has so long existed. It leads the way to subsequent quantitative analysis and even now provides some basis for comparing investments in land reclamation and fertilizer production.

Fertilizer substitutes for land in the sense that a given product can be produced with less land and added fertilizer on the remaining

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land. Fertilizer also substitutes for labour since it boosts the yield but increases labour requirements by a very small absolute amount and as a minute fraction of (*a*) the total labour used per acre and (*b*) the relative increase in per acre yield. A given yield, under the use of more fertilizer, can therefore be obtained with less of both land and labour.

This paper presents empirical estimates of "gross" marginal rates of substitution between fertilizer in aggregate form (*i.e.*, a given mix of nutrients), land and labour. These substitution rates are "gross" because the other minor capital items, as well as labour associated with fertilizer application per acre yield increase, are not included in this study. The estimates of substitution rates refer to particular types of land locations, climate and other environmental factors. With subsequent knowledge over wider experimental data, more information can be obtained for the potential substitution rates between wider ranges of resources. This knowledge is not only useful in developing countries where land is severely restricted but also in planning national policies for developed nations.

We present substitution rates derived from two different algebraic forms of production function. The quadratic and square-root models analysed are widely used in fertilizer production functions studies. The substitution rates derived from applying these two models fitted to experimental data are compared for estimational suitability and practical usefulness.

Derivation of Substitution Rates for Land

The method of deriving "gross" marginal rates of substitution from experimental production functions has been explained by Heady (2) and has been used in other studies (3, 4). However, the procedure is reviewed briefly with modifications. A particular production function derived from a fixed land area with variable fertilizer inputs can be extended to reflect variable land quantities and substitution rates between land and fertilizer. While particular production function is related initially to fertilizer response, it can be suitably transformed, through some mathematical conversions, into one that includes land and has "constant returns to scale" for the two factors considered alone, so that doubling of both land and fertilizer will double output.

Quadratic Model : Equation 1, a quadratic model, is one of the two types of algebraic models examined :

$$Y=a+bN+cP-dN^2-eP^2+fNP \qquad ...(1)$$

N and P denote nitrogen and phosphorus in pounds per acre, respectively, and Y denotes output per acre. Many proportions or mixes of N and P can be derived from this equation. For example, a mix equal to the proportions of nutrients historically used or recommended at the location of the data can be used. In our study, however, the mix is varied between two values containing the recommended optimum ratio.

The conversion is as follows where r units of N are specified for each unit of P or $P=rN$ to produce one unit of F or fertilizer. With F, N and P all measured in pound units, the following relations are obvious :

$$F=(1+r)N \qquad \text{...(2)}$$

$$N=\frac{F}{r+1} \qquad \text{...(3)}$$

$$P=\frac{rF}{r+1} \qquad \text{...(4)}$$

Substituting these values of N and P in equation 1 and simplifying we obtain

$$Y=a+BF-CF^2 \qquad \text{...(5)}$$

where $$B=\frac{b+cr}{r+1}$$

and $$C=\frac{d+er^2-fr}{(r+1)}$$

Equation 5 represents a "per acre" production function where F now is the fertilizer and Y is the output per acre. A more exact form of equation 5 can be written as

$$\frac{Y}{A}=a+B\left(\frac{F}{A}\right)-C\left(\frac{F}{A}\right)^2 \qquad \text{...(6)}$$

Where A is land in acres. Here the output per acre $\left(\frac{Y}{A}\right)$ is a function of total fertilizer input per acre $\left(\frac{F}{A}\right)$. Multiplying both sides of equation 6 by A to allow land also to be a variable, we have

$$Y=aA+BF-CF^2A^{-1} \qquad \text{...(7)}$$

Here the total output is expressed as a function of A input of land in acres and F input of fertilizer in pounds. The *per acre* production function with land fixed in equation 6 is thus transformed to the "long run" function as in (7) with total output Y as a function of variable land inputs A and total fertilizer inputs F. If the number of acres and the amount of fertilizer are increased by a given proportion in equation 7, then the total output increases by the same proportion. This assumption of constant returns to scale arises from the procedure of estimating production functions under experimental conditions. Equation 7, embodies the assumption that each land input contains the "fixed" experimental conditions including soil types, rainfall, temperature, seed, etc. Similarly each fertilizer input is accompanied with appropriate labour and other inputs necessary for applying fertilizer. Under these conditions, constant returns to scale in equation 7 is a reasonable assumption.

Solving equation 7, the isoquant equation is obtained as

$$A=\frac{(Y-BF)+\sqrt{(Y-BF)^2+4aCF^2}}{2a} \qquad ...(8)$$

The isoquant equation indicates the various combinations of land and fertilizer that will produce a given output Y. The negative ratio of partial derivatives with respect to F and A from equation 7 is as follows :

$$\frac{dA}{dF}=\frac{2CAF-BA^2}{aA^2+CF^2} \qquad ...(9)$$

Equation 9 defines the gross marginal rate of substitution of fertilizer (F) for land (A). The term "gross" is used because, as indicated earlier, "fixed" inputs such as seed, plows, labour, etc., are associated with land A and "variable" inputs such as additional labour and capital required to apply fertilizer are included with F.

Square-root Model : With the same inputs as N and P, the square-root model considered is the second model examined :

$$Y=a_1+b_1N^{\frac{1}{2}}+c_1P^{\frac{1}{2}}-d_1N-e_1P+f_1N^{\frac{1}{2}}P^{\frac{1}{2}} \qquad ...(10)$$

Using the conversion $P=rN$ and substituting the values of N and P from equations 3 and 4 into equation 10 and simplifying, we abtain

$$Y=a_1+B_1F^{\frac{1}{2}}-C_1F \qquad ...(11)$$

where $$B_1=\frac{b_1+c_1r^{\frac{1}{2}}}{(r+1)^{\frac{1}{2}}} \text{ and } C_1=\frac{d_1+e_1r-f_1r^{\frac{1}{2}}}{r+1}$$

Introducing A, the land in acres into equation 11, we have :

$$Y=a_1A+B_1F^{\frac{1}{2}}\ A^{\frac{1}{2}}-C_1F \qquad ...(12)$$

Under the conditions specified earlier, equation 12 also embodies "constant returns to scale". Solving equation 12, the isoquant equation is :

$$A^{\frac{1}{2}}=\frac{-B_1F^{\frac{1}{2}}+\sqrt{B_1{}^2F+4a_1(C_1F+Y)}}{2a_1} \qquad ...(13)$$

It indicates the various combinations of land and fertilizer to produce a given output Y under the square-root model. The gross marginal rate of substitution of fertilizer for land is now :

$$\frac{dA}{dF}=\frac{2C_1A^{\frac{1}{2}}F^{\frac{1}{2}}-B_1A}{2a_1A^{\frac{1}{2}}F^{\frac{1}{2}}+B_1F} \qquad ...(14)$$

Substitution Rates for Labour

Since fertilizer is also a substitute for labour, the marginal rate of substitution of fertilizer for labour can be defined somewhat similarly For purposes of this paper, land and labour are considered as technical complements with k units of labour used for one acre of land. Under other formulations, land and labour are substitutes in food production. However, the increment of labour used in applying fertilizer and in harvesting the added yield is small compared to the total per acre labour requirements. Hence, land and labour are considered held in fixed proportions such that $L=kA$ or $A=k^{-1}L$ where L is 8-hour days of labour used and k is 8-hour days of labour required per acre. Bullock labour also is an important contributing factor in agricultural production of India. Hence, it is added to human labour in computing values for L and k and, k, the labour days required per acre, includes both human and bullock labour. Human labour is measured in 8-hour days whereas bullock labour is measured in 8-hour pair days as bullocks are used usually in pairs.

Quadratic Model: Substituting $A=k^{-1}L$ in equation 7, we obtain equation 15, where output is a function of the amount of labour and fertilizer used, based grossly on simple experiments

$$Y=ak^{-1}L+BF-CkL^{-1}F^2 \qquad ...(15)$$

and B and C are as defined in equation 5.

Solving equation 15, the isoquant equation is :

$$L=\frac{(Y-BF)k+\sqrt{(Y-BF)^2k^2+4aCF^2k^2}}{2a} \qquad ...(16)$$

This isoquant equation indicates the various combinations of labour and fertilizer that will produce a given output Y (supposing land to vary directly with labour). From equation 15, we derive

$$\frac{dL}{dF}=\frac{2Ck^2LF-BkL^2}{aL^2+Ck^2F^2} \qquad ...(17)$$

Equation 17 defines the gross marginal rate of substitution of fertilizer (F) for labour (L).

Square-root Model : Substituting $A=k^{-1}L$ in equation 12 we obtain the production function, isoquant equation and marginal rate of substitution, respectively, in equations 18, 19 and 20 where B_1 and C_1 are as defined in equation 11.

$$Y=a_1k^{-1}L+B_1F^{\frac{1}{2}}k^{-\frac{1}{2}}L^{\frac{1}{2}}-C_1F \qquad ...(18)$$

$$L^{\frac{1}{2}}=\frac{-B_1F^{\frac{1}{2}}k^{\frac{1}{2}}+\sqrt{B_1^2Fk+4a_1(C_1F+Y)k}}{2a_1} \qquad ...(19)$$

$$\frac{dL}{dF}=\frac{2C_1kF^{\frac{1}{2}}L^{\frac{1}{2}}-B_1k^{\frac{1}{2}}L}{2a_1F^{\frac{1}{2}}L^{\frac{1}{2}}+Fk^{\frac{1}{2}}} \qquad ...(20)$$

Empirical Estimates of Substitution of Fertilizer for Land

The substitution rates derived are not predictions of those which have existed in Indian agriculture. Instead, they represent substitution rates under the specific environmental conditions of the data. Thus they indicate a specific set of physical potentials. The sample of functions considered is not necessarily representative for the state concerned in respect to soils and similar phenomena. The functions are used because of their availability and because they satisfy certain economic criteria for which they were fitted. While the statistical data are entirely meaningful in deriving the production functions of the quadratic and square-root forms, the purpose here is not to predict for time and the nation but to indicate potential substitution rates under specific conditions.

All estimates are for paddy. Derivation of gross marginal rates of substitution are made for two locations: (1) Tirurkuppam, Madras for the year 1948-49, and (2) Chandukuri, Madhya Pradesh for the year 1936-37. The basic production functions are reported by Abraham and Rao (1). However, some results are presented in the appendix to give the reader the useful background information.

For convenience, only four isoquants are considered for each function and location, all the isoquants representing a yield level obtainable on a single acre but not restricted to an acre as a fixed input magnitude. The isoquant levels are so chosen to represent a range of production, keeping in view the optimum values of outputs (appendix). Thus the isoquant levels selected are 10, 15, 20 and 25 maunds (1 maund = 82·2857 pounds) per acre of paddy. Also, the substitution rates are derived when r, the ratio of P to N takes values 0·5, 1·0 and 1 5. Though r can take any positive finite value, these three values are selected for r in the light of information regarding the optimum fertilizer combinations given in the appendix.

The estimated quantities for isoquants and 'gross' marginal rates of substitution for fertilizer and land are given in Tables 1 and 2 for Tirurkuppam and Chandukuri, respectively. As expected, the substitution rates of fertilizer for land increase in absolute value with the higher yield isoquants for any value of r, the ratio of P to N. Also, for any selected isoquant level and for any given value of r, the marginal rates of substitution decrease as the fertilizer (F) rate increases. When no fertilizer is applied, the marginal rates of substitution are infinite for the square-root. Also, the substitution rates of fertilizer for land increase in absolute value with the higher yield isoquants for any ratio of P to N. Also for any selected isoquant level and for any given value of r, the marginal rates of substitution decrease as the fertilizer (F) rate increases.

Under the quadratic model when r is equal to unity, a 15 maunds output for Tirurkuppam (Table 1) location is obtained with 2·06 acres of land and no fertilizer, 1·81 acres of land and 10 pounds of fertilizer, 1·58 acres of land and 20 pounds of fertilizer, 1·19 acres of land and 40 pounds of fertilizer, etc. With the combination of 20 pounds of fertilizer and 1·58 acres of land for 15 maunds output, a pound of fertilizer substitutes for 0·0220 acres of land. Hence a ton of fertilizer nutrients similarly spread over more acres of land is estimated to substitute for 49·28 acres of land (*i.e.*, $2240 \times 0{\cdot}0220$). With 60 pounds of fertilizer and 0 94 acres of land to produce the same amount of paddy, a ton of fertilizer substitutes for 19 94 acres of land. With 60 pounds of fertilizer and 1·49 acres of land to produce 20 maunds of paddy, a ton of fertilizer substitutes for 32·92 acres of land.

When r is equal to 1·00 under the square-root model, 15 maunds of output for the same location is forthcoming with 1·73 acres of land and no fertilizer, 1·35 acres of land and 10 pounds of fertilizer, 1·22 acres of land and 20 pounds of fertilizer, etc. With the combination of 20 maunds of fertilizer and 1·22 acres of land for a 15 maunds output, a ton of fertilizer substitutes for 24·64 acres. With 60 pounds of fertilizer and 0·93 acres of land to produce the same amount of output, a ton of fertilizer substitutes for 11·20 acres of land. With 60 pounds of fertilizer 1·35 acres of land to produce 20 maunds of paddy, a ton of fertilizer substitutes for 13·89 acres of land.

For Chandukuri (Table 2) location when r is equal to 1·00 under the quadratic model, 20 pounds of fertilizer and 1·05 acres of land produce 15 maunds of paddy. Hence a ton of fertilizer substitutes for 41·22 acres of land. With 60 pounds of fertilizer and 0·96 acres of land to produce 20 maunds of paddy, a ton of fertilizer substitutes for 17·25 acres of land.

When r is equal to 1·00 under the square-root model, 20 pounds of fertilizer and 0·88 acres of land produce 15 maunds of paddy. Hence, a ton of fertilizer substitutes for 20·61 acres of land. With 60 pounds of fertilizer and 0·68 acres of land to produce 15 maunds of output, a ton of fertilizer substitutes for 6·7 acres of land. With 60 pounds of fertilizer and 0·97 acres of land to produce 20 maunds of paddy, a ton of fertilizer substitutes for 9·41 acres of land.

Obviously, the gross marginal rate of substitution of fertilizer nutrients for land varies with the soil type, rainfall, crop, climate and other environmental factors—as well as with the ratios in which fertilizer and land are combined under any unique combination of these factors. The rate at which fertilizer substitutes for land also varies with the level of fertilization of each acre of land.

The land-fertilizer isoquants for paddy derived from both models are illustrated in Figures 1 and 2 when $r=1$ for Tirurkuppam and Chandukuri locations, respectively. The slopes of these isoquants define the gross marginal rates of substitution between fertilizer and land. The slopes of the isoquants of the quadratic function decline

TABLE 2

*Isoquants and "Gross" Marginal Rates of Substitution (MRS) of Fertilizer ($F = N + P$) for Land (A) — Chandukuri**

$r = \frac{P}{N}$	F lbs. per acre	QUADRATIC MODEL								SQUARE-ROOT MODEL							
		10 maunds		15 maunds		20 maunds		25 maunds		10 maunds		15 maunds		20 maunds		25 maunds	
		A	*MRS*	*A*	*MRS*	*A*	*MRS*	*A*	*MRS*	*A*	*MRS*	*A*	*MRS*	*A*	*MRS*	*A*	*MRS*
$r = 0.50$	0	0.98	−0.0178	1.47	−0.0178	1.96	−0.0178	2.45	−0.0178	0.94	∞	1.42	∞	1.89	∞	2.36	∞
	10	0.81	−0.0164	1.30	−0.0169	1.78	−0.0172	2.27	−0.0173	0.65	−0.0122	1.05	−0.0159	1.46	−0.0191	1.87	−0.0291
	20	0.66	−0.0139	1.12	−0.0157	1.62	−0.0164	2.10	−0.0167	0.56	−0.0074	0.93	−0.0099	1.31	−0.0122	1.70	−0.0141
	40	0.45	−0.0060	0.86	−0.0115	1.31	−0.0139	1.78	−0.0150	0.45	−0.0042	0.77	−0.0059	1.12	−0.0074	1.48	−0.0087
	60	0.40	−0.0007	0.68	−0.0060	1.07	−0.0101	1.51	−0.0125	0.38	−0.0029	0.68	−0.0042	1.00	−0.0053	1.33	−0.0064
	80	—		0.61	−0.0019	0.91	−0.0060	1.29	−0.0093	0.33	−0.0022	0.60	−0.0032	0.90	−0.0042	1.22	−0.0051
	100	—		—		0.82	−0.0027	1.14	−0.0060	0.29	−0.0017	0.54	−0.0026	0.82	−0.0034	1.13	−0.0042
	120	—		—		0.79	−0.0007	1.04	−0.0033	0.26	−0.0014	0.50	−0.0021	0.76	−0.0029	1.05	−0.0036
$r = 1.00$	0	0.98	−0.0227	1.47	−0.0227	1.96	−0.0227	2.45	−0.0227	0.94	∞	1.42	∞	1.89	∞	2.36	∞
	10	0.76	−0.0199	1.25	−0.0210	1.74	−0.0215	2.23	−0.0218	0.62	−0.0117	1.00	−0.0161	1.40	−0.0198	1.80	−0.0232
	20	0.59	−0.0146	1.05	−0.0184	1.53	−0.0199	2.01	−0.0206	0.54	−0.0064	0.88	−0.0092	1.25	−0.0117	1.62	−0.0140
	40	0.43	−0.0026	0.76	−0.0100	1.18	−0.0146	1.64	−0.0170	0.45	−0.0030	0.75	−0.0048	1.08	−0.0064	1.42	−0.0078
	60	—		0.64	−0.0026	0.96	−0.0077	1.34	−0.0108	0.40	−0.0018	0.68	−0.0030	0.97	−0.0042	1.29	−0.0053
	80	—		—		0.86	−0.0026	1.16	−0.0065	0.37	−0.0012	0.62	−0.0021	0.90	−0.0030	1.20	−0.0039
	100	—		—		—		1.07	−0.0026	0.35	−0.0008	0.59	−0.0016	0.85	−0.0023	1.13	−0.0030
	120	—		—		—		1.05	−0.0003	0.30	−0.0006	0.56	−0.0012	0.81	−0.0018	1.07	−0.0024
$r = 1.50$	0	0.98	−0.0256	1.47	−0.0256	1.96	−0.0256	2.45	−0.0256	0.94	∞	1.42	∞	1.89	∞	2.36	∞
	10	0.74	−0.0213	1.22	−0.0231	1.71	−0.0239	2.20	−0.0243	0.61	−0.0111	0.99	−0.0158	1.37	−0.0198	1.77	−0.0234
	20	0.57	−0.0133	1.01	−0.0190	1.48	−0.0213	1.96	−0.0224	0.54	−0.0056	0.87	−0.0085	1.23	−0.0111	1.59	−0.0135
	40	0.45	−0.0003	0.75	−0.0075	1.12	−0.0122	1.57	−0.0169	0.46	−0.0023	0.76	−0.0040	1.07	−0.0056	1.40	−0.0071
	60	—		0.68	−0.0003	0.95	−0.0051	1.30	−0.0097	0.43	−0.0012	0.69	−0.0023	0.98	−0.0034	1.29	−0.0045
	80	—		—		0.90	−0.0003	1.17	−0.0038	0.41	−0.0006	0.66	−0.0014	0.93	−0.0023	1.21	−0.0032
	100	—		—		—		1.13	−0.0034	0.40	−0.0003	0.63	−0.0009	0.89	−0.0016	1.16	−0.0023
	120	—		—		—		—		0.40	−0.0001	0.62	−0.0006	0.86	−0.0012	1.12	−0.0017

* *A* refers to the amount of land to produce the specified yield per acre when fertilizer is at the level in the *F* column to the left. The *MRS* is the marginal rate of substitution of fertilizer per acre when the combination of the two inputs is that shown in the *F* column and the *A* column for a particular yield level.

TABLE 1

*Isoquants and "Gross" Marginal Rates of Substitution (MRS) of Fertilizer ($F = N + P$) for Land (A) — Tirurkuppam**

$r = \frac{P}{N}$	F lbs. per acre	QUADRATIC MODEL								SQUARE-ROOT MODEL							
		10 maunds		15 maunds		20 maunds		25 maunds		10 maunds		15 maunds		20 maunds		25 maunds	
		A	*MRS*	*A*	*MRS*	*A*	*MRS*	*A*	*MRS*	*A*	*MRS*	*A*	*MRS*	*A*	*MRS*	*A*	*MRS*
	0	1.37	−0.0205	2.06	−0.0205	2.74	−0.0205	3.43	−0.0205	1.16	∞	1.73	∞	2.31	∞	2.89	∞
	10	1.17	−0.0197	1.85	−0.0200	2.54	−0.0201	3.22	−0.0202	0.87	−0.0135	1.39	−0.0168	1.91	−0.0196	2.44	−0.0220
	20	0.98	−0.0183	1.66	−0.0193	2.34	−0.0197	3.02	−0.0199	0.76	−0.0092	1.25	−0.0115	1.75	−0.0135	2.26	−0.0158
	40	0.66	−0.0218	1.29	−0.0169	1.96	−0.0183	2.63	−0.0189	0.62	−0.0060	1.06	−0.0078	1.53	−0.0092	2.01	−0.0104
$r = 0.50$	60	0.48	−0.0053	0.99	−0.0128	1.61	−0.0161	2.27	−0.0175	0.51	−0.0046	0.93	−0.0060	1.37	−0.0072	1.82	−0.0083
	80	0.42	−0.0012	0.79	−0.0076	1.32	−0.0128	1.94	−0.0155	0.43	−0.0038	0.82	−0.0050	1.23	−0.0060	1.67	−0.0070
	100	—		0.68	−0.0035	1.10	−0.0089	1.65	−0.0128	0.36	−0.0032	0.72	−0.0043	1.12	−0.0052	1.54	−0.0060
	120	—		0.64	−0.0012	0.96	−0.0053	1.43	−0.0097	0.30	−0.0027	0.64	−0.0038	1.02	−0.0046	1.43	−0.0054
	0	1.37	−0.0248	2.06	−0.0248	2.74	−0.0248	3.43	−0.0248	1.16	∞	1.73	∞	2.31	∞	2.89	∞
	10	1.13	−0.0229	1.81	−0.0237	2.50	−0.0240	3.18	−0.0242	0.85	−0.0132	1.35	−0.0171	1.86	−0.0203	2.38	−0.0232
	20	0.92	−0.0197	1.58	−0.0220	2.26	−0.0229	2.94	−0.0234	0.75	−0.0083	1.22	−0.0110	1.70	−0.0132	2.20	−0.0152
	40	0.62	−0.0089	1.19	−0.0165	1.83	−0.0197	2.50	−0.0212	0.62	−0.0050	1.04	−0.0068	1.49	−0.0083	1.96	−0.0097
$r = 1.00$	60	0.53	−0.0014	0.94	−0.0089	1.49	−0.0147	2.10	−0.0179	0.54	−0.0036	0.93	−0.0050	1.35	−0.0062	1.79	−0.0073
	80	—		0.82	−0.0031	1.25	−0.0089	1.79	−0.0135	0.47	−0.0028	0.84	−0.0039	1.24	−0.0050	1.66	−0.0059
	100	—		0.79	−0.0002	1.12	−0.0043	1.56	−0.0089	0.42	−0.0022	0.77	−0.0032	1.15	−0.0042	1.55	−0.0050
	120	—		—		1.07	−0.0014	1.42	−0.0051	0.38	−0.0019	0.71	−0.0028	1.07	−0.0036	1.46	−0.0043
	0	1.37	−0.0274	2.06	−0.0274	2.74	−0.0274	3.43	−0.0274	1.16	∞	1.73	∞	2.31	∞	2.89	∞
	10	1.11	−0.0244	1.79	−0.0256	2.47	−0.0261	3.16	−0.0264	0.84	−0.0126	1.34	−0.0167	1.84	−0.0203	2.36	−0.0232
	20	0.89	−0.0193	1.55	−0.0230	2.22	−0.0244	2.90	−0.0252	0.75	−0.0075	1.21	−0.0102	1.69	−0.0126	2.18	−0.0147
	40	0.64	−0.0057	1.16	−0.0146	1.78	−0.0193	2.43	−0.0216	0.64	−0.0041	1.06	−0.0059	1.50	−0.0075	1.95	−0.0089
$r = 1.50$	60	—		0.96	−0.0057	1.46	−0.0122	2.04	−0.0166	0.57	−0.0029	0.96	−0.0041	1.37	−0.0053	1.80	−0.0065
	80	—		0.91	−0.0006	1.29	−0.0057	1.77	−0.0107	0.52	−0.0020	0.89	−0.0031	1.28	−0.0041	1.69	−0.0050
	100	—		—		1.22	−0.0015	1.61	−0.0057	0.49	−0.0016	0.83	−0.0025	1.20	−0.0033	1.60	−0.0041
	120	—		—		—		1.53	−0.0022	0.46	−0.0012	0.79	−0.0020	1.14	−0.0029	1.52	−0.0035

* *A* refers to amount of land to produce the specified yield per acre when fertilizer is at the level in the *F* column to the left. The *MRS* is the marginal rate of substitution of fertilizer per acre when the combination of the two inputs is that shown in the *F* column and the *A* column for a particular yield level.

more or less steadily as the fertilization rate increases. But the slopes of the isoquants of square-root function decline very rapidly at lower rates of fertilization. As the fertilization rate is increased, the rate of decline is reduced. The isoquant lines for the square-root form are lower than those of quadratic form at lower fertilization rates Hence, at lower fertilization rates—these rates increase from 30 to 60 pounds per acre as the isoquant level increases from 10 to 25 maunds per acre of paddy—isoquant yields under the square-root model are obtained with less of land and less of labour than those required under quadratic model. When the slopes of these isoquants are zero, it indicates that no land can be replaced by fertilizer.

For the isoquant of 20 maunds per acre when r is allowed to take three different values, the marginal rates of substitution for Tirurkuppam are illustrated in Figures 3 and 4 for quadratic and square-root models, respectively. For the quadratic model (Figure 3), at the lower rates of fertilization, the marginal rates of substitution of fertilizer for land increase in absolute value as the value of r increases. However, these substitution rates decrease as the value of r increases at higher levels of fertilization. As r increases from 0.50 to 1.50, the substitution rates increase as the fertilization rate increases up to 30 pounds per acre. But beyond 60 pounds of fertilization the rates decrease as the value of r increases. For the square-root model (Figure 4), the substitution rates are infinite when no fertilizer is applied. Up to 10 pounds of fertilization, these rates do not differ much as the value of r changes. But at higher rates of fertilization, the substitution rates decrease as the value of r increases. For any given isoquant, the differences in marginal rates of substitution for varying values of r are much greater for the quadratic function than the square-root function.

When $r=1$ and when the isoquant level is 20 maunds per acre, the marginal rates of substitution for both models are illustrated in Figures 5 and 6 for Tirurkuppam and Chandukuri, respectively. At two levels (one at high and the other at low) of fertilization, the substitution rates derived from both models are identical. At very low levels of fertilization, *i e*., at 10 pounds of fertilizer per acre or lower, the substitution rates derived from square-root model are extremely high and hence are not meaningful. Otherwise, the substitution rates derived from the square-root form are generally lower than those derived form the quadratic form for relevant ranges of fertilizer application.

Substitution of Fertilizer for Labour

Marginal rates of substitution of fertilizer for labour are derived from previous equations and data. Here, labour is marginally associated with land, in the sense that if we replace an acre of land by fertilizing remaining acres at a higher level, we also displace the constant quantity of labour required to handle the "displaced" land. As a given output is produced by diverting some land from production and producing more on fewer acres at a higher yield, some of the displaced labour (attached to the displaced land) is offset by the added labour required to harvest and handle the higher yield on the remaining acres, as well as by some added labour for applying the fertilizer. However, under Indian farming conditions, the incremental labour to apply fertilizer and harvest the greater yield is very small to the total labour repuirements and hence may possibly be neglected in the aggregate importance. Or, one can recognize the substitution rates for labour presented here are slightly higher than the actual "net" rates.

Again, the rates of substitution of fertilizer for labour depend on environmental conditions at each location, as well as on the proportions in which labour and fertilizer are combined. These rates are calculated for the same isoquant levels and the same values of r that are used in the earlier section. However, the national average labour requirements per acre of paddy are not available. Panse and Bokil (5) estimated labour input quantities per hectare for selected crops and zones of the nation. The estimates used in this paper are the averages of three years 1954-55 to 1956-57. The estimates for Madras zone are used for Tirurkuppam. Converting the figures to a per acre basis for Madras, it is estimated that 109.3 eight-hour days of human labour and 55.8 eight-hour pairs days of bullock labour are needed for one acre of paddy. Hence the value of k for Tirurkuppam is taken at 165.1 eight-hour days which contains 66 per cent of human labour and 34 per cent of bullock labour. The estimates for Madhya Pradesh zone are taken for Chandukuri. Converting the figures to per acre basis, 36 8 eight-hour days of human labour and 8.9 eight-hour pair days of bullock labour are needed for the zone. Accordingly the value of k for Chandukuri is considered at 45.7 eight-hour days which includes 88 per cent of human labour and 20 per cent of bullock labour. Higher inputs of both human and bullock labour in Madras region are explained as a consequence of the use of lift irrigation in the region.

The substitution rates for labour are presented in Tables 3 and 4 for Tirurkuppam and Chandukuri locations, respectively. (Graphic

TABLE 3

*Isoquants and "Gross" Marginal Rates of Substitution (MRS) of Fertilizer ($F = N + P$) for Labour (L) — Tirurkuppam**

$r = \frac{P}{N}$	F lbs. per acre	QUADRATIC MODEL								SQUARE-ROOT MODEL							
		10 maunds		15 maunds		20 maunds		25 maunds		10 maunds		15 maunds		20 maunds		25 maunds	
		L	*MRS*	*L*	*MRS*	*L*	*MRS*	*L*	*MRS*	*L*	*MRS*	*L*	*MRS*	*L*	*MRS*	*L*	*MRS*
	0	226.3	−3.387	339.5	−3.387	452.6	−3.387	565.8	−3.387	190.8	∞	286.2	∞	381.6	∞	477.0	∞
	10	193.1	−3.246	306.0	−3.300	419.1	−3.324	532.2	−3.339	144.4	−2.137	228.9	−2.680	315.2	−3.137	402.6	−3.539
	20	161.7	−3.020	273.6	−3.183	386.2	−3.246	499.1	−3.280	126.2	−1.426	206.3	−1.813	288.7	−2.137	372.8	−2.423
	40	109.2	−2.116	213.4	−2.795	323.4	−3.020	435.0	−3.124	101.9	−0.918	175.4	−1.195	252.5	−1.426	331.7	−1.630
$r = 0.50$	60	79.6	−0.879	163.8	−2.116	266.3	−2.655	374.6	−2.894	84.5	−0.689	152.9	−0.918	225.7	−1.109	301.2	−1.276
	80	69.9	−0.199	130.0	−1.256	218.4	−2.116	319.9	−2.561	70.7	−0.550	134.7	−0.751	203.9	−0.918	276.2	−1.064
	100	——		112.2	−0.582	182.5	−1.466	272.9	−2.116	59.4	−0.454	119.4	−0.636	185.3	−0.786	254.8	−0.918
	120	——		104.8	−0.199	159.2	−0.879	235.8	−1.597	49.8	−0.382	106.1	−0.550	169.0	−0.689	236.0	−0.809
	0	226.3	−4.098	339.5	−4.098	452.6	−4.098	565.8	−4.098	190.8	∞	286.2	∞	381.6	∞	477.0	∞
	10	186.8	−3.787	299.4	−3.911	412.3	−3.964	525.3	−3.994	140.5	−2.145	223.1	−2.774	307.7	−3.308	393.6	−3.780
	20	151.3	−3.252	261.6	−3.639	373.5	−3.787	486.0	−3.864	123.4	−1.340	200.8	−1.774	281.0	−2.146	363.0	−2.475
	40	103.2	−1.474	197.0	−2.731	302.6	−3.252	412.1	−3.500	102.4	−0.793	172.7	−1.086	246.8	−1.340	323.3	−1.567
$r = 1.00$	60	88.1	−0.229	154.8	−1.474	245.4	−2.428	347.2	−2.955	88.5	−0.563	153.6	−0.793	223.1	−0.994	295.6	−1.174
	80	——		135.8	−0.519	206.4	−1.474	295.1	−2.237	78.2	−0.434	139.0	−0.624	204.8	−0.793	273.0	−0.946
	100	——		130.9	−0.029	185.1	−0.708	258.1	−1.474	69.9	−0.349	127.1	−0.513	189.7	−0.659	255.9	−0.793
	120	——		——		176.2	−0.230	235.3	−0.837	63.2	−0.290	117.2	−0.434	177.0	−0.563	240.7	−0.682
	0	226.3	−4.525	339.5	−4.525	452.6	−4.525	565.8	−4.525	190.8	∞	286.2	∞	381.6	∞	477.0	∞
	10	183.3	−4.035	295.6	−4.232	408.4	−4.316	521.3	−4.363	139.5	−2.068	221.1	−2.735	304.9	−3.304	390.0	−3.810
	20	146.8	−3.183	255.3	−3.796	366.6	−4.035	478.7	−4.157	123.6	−1.226	199.8	−1.677	278.9	−2.068	360.0	−2.417
	40	106.3	−0.939	192.2	−2.416	293.7	−3.183	401.0	−3.573	105.5	−0.672	174.5	−0.965	247.2	−1.226	322.5	−1.461
$r = 1.50$	60	——		159.4	−0.939	241.5	−2.011	337.5	−2.735	94.4	−0.450	158.2	−0.672	226.4	−0.872	297.6	−1.056
	80	——		150.1	−0.097	212.6	−0.939	292.4	−1.772	86.6	−0.329	146.4	−0.507	211.0	−0.672	278.8	−0.824
	100	——		——		201.3	−0.251	265.7	−0.939	80.7	−0.253	137.2	−0.402	198.7	−0.541	263.7	−0.672
	120	——		——		——		253.1	−0.359	76.1	−0.202	129.8	−0.329	188.7	−0.450	251.2	−0.564

**L* refers to the amount of labour to produce the specified yield per acre when fertilizer is at the level in the *F* column to the left. The *MRS* is the marginal rate of substitution of fertilizer per acre when the combination of the two inputs is that shown in the *F* column and *L* column for a particular yield level.

TABLE 4

Isoquants and "Gross" Marginal Rates of Substitution (MRS) of Fertilizer ($F = N+P$) for Ladour (L) — Chanduk

$r = \frac{P}{N}$	F lbs. per acre	QUADRATIC MODEL								SQUARE-ROOT MODEL							
		10 maunds		15 maunds		20 maunds		25 maunds		10 maunds		15 maunds		20 maunds		25 maunds	
		L	*MRS*	*L*	*MRS*	*L*	*MRS*	*L*	*MRS*	*L*	*MRS*	*L*	*MRS*	*L*	*MRS*	*L*	*MRS*
$r = 0.60$	0	44.8	−0.814	67.2	−0.814	89.6	−0.814	112.0	−0.814	43.2	∞	64.8	∞	86.4	∞	108.1	∞
	10	36.9	−0.748	59.2	−0.774	81.6	−0.786	103.9	−0.792	29 9	−0.572	48.0	−0 744	66.6	−0.891	85 6	−1.021
	20	30.0	−0.634	51.7	−0.716	73.9	−0.748	96 1	−0.764	25.6	−0.352	42.3	−0.470	59.8	−0.572	77.7	−0.662
	40	20.8	−0.273	39.2	−0.526	60.0	−0.634	81.6	−0.687	20.6	−0.203	35.4	−0.282	51.2	−0.351	67.7	−0.413
	60	18.1	−0.034	31.2	−0.273	48.9	−0.463	68 9	−0.572	17.4	−0.141	30.9	−0.203	45.5	−0.257	60.9	−0.306
	80	—		27.7	−0.089	41 6	−0.273	58.6	−0.424	15.1	−0.107	27.5	−0.158	41.2	−0.203	55.7	−0.244
	100	—		—		37.7	−0.125	52 0	−0.273	13.4	−0.085	24 9	−0.128	37.7	−0.167	51.5	−0.203
	120	—		—		36.2	−0.034	47.8	−0.150	12.0	−0.070	22.7	−0.107	34.8	−0.141	47.9	−0.173
$r = 1.00$	0	44 8	−1.038	67.2	−1.038	89.6	−1.038	112.0	−1.038	43.2	∞	64.8	∞	86.4	∞	108.1	∞
	10	35.0	−0.908	57.2	−0.962	79.4	−0.984	101.8	−0.996	28.5	−0.574	45.9	−0.778	63.9	−0.954	82.4	−1.110
	20	27.0	−0.666	48.1	−0.841	70.0	−0 908	92.1	−0 942	24 6	−0.321	40,4	−0.456	57.0	−0.574	74.2	−0.681
	40	19.6	−0.119	34.9	−0.456	54.0	−0.666	74.8	−0 778	20.6	−0.159	34.4	−0.244	49.2	−0.321	64.8	−0.391
	60	—		29.5	−0.119	43.8	−0 354	61.5	−0.542	18.4	−0.098	30.9	−0,159	44.5	−0.217	58.9	−0.270
	80	—		—		39.3	−0.119	53.1	−0.297	17.1	−0.066	28.6	−0.114	41.2	−0.159	54.7	−0.203
	100	—		—		—		49.1	−0.119	16.2	−0.046	26.9	−0.085	38.8	−0.123	51.5	−0.159
	120	—		—		—		47.9	−0.013	15.5	−0.034	25.6	−0.066	36.9	−0.098	49.0	−0.129
$r = 1.50$	0	44 8	−1.172	67.2	−1.172	89.6	−1.172	112.0	−1.172	43.2	∞	64.8	∞	86.4	∞	108.1	∞
	10	33.9	−0.973	56.0	−1.057	78.2	−1.091	100.5	−1.110	28.0	−0.558	45.1	−0.776	62.8	−0.966	80.0	−1.134
	20	25.9	−0.610	46.3	−0.870	67.9	−0.973	89.8	−1.026	24.4	−0 290	39.8	−0.432	56 1	−0.558	72.9	−0.671
	40	20.7	−0.016	34.1	−0.342	51.8	−0.610	71.7	−0 772	21.2	−0.125	34.5	−0.211	48.9	−0.290	64.0	−0.363
	60	—		31.0	−0 016	43.6	−0.232	59.5	−0.444	19.6	−0.065	31.7	−0.125	44.9	−0.183	58.8	−0.238
	80	—		—		41.4	−0.016	53.5	−0.175	18.9	−0.034	30.0	−0.080	42.3	−0.125	55.4	−0.169
	100	—		—		—		51.7	−0.016	18.5	−0.016	29.0	−0.052	40.5	−0.089	52.9	−0.125
	120	—		—		—		—		18.3	−0.004	28.3	−0.034	39.3	−0.065	51.1	−0.095

**L* refers to the amount of labour to produce the specified yield per acre when fertilizer is at the level in the *F* column to the left. The *MRS* is the marginal rate of substitution of fertilizer per acre when the combination of the two inputs is that shown in the *F* column and *L* column for a particular yield level.

representation is not included since the labour isoquants have the same configuration as those for land at the different locations and for the different equations.) When $r=1$ for Tirurkuppam (Table 3) under quadratic model, 15 maunds of output is forthcoming either with no fertilizer and 339.5 eight-hour days of labour or with 20 pounds of fertilizer and 261.6 days of labor or with 100 pounds of fertilizer and 130.9 days of labour. Hence with zero level of fertilizer, one pound of fertilizer substitutes for 4.10 days of labour. With 20 pounds of fertilizer, one pound of fertilizer substitutes for 3.64 days of labour. And with 100 pounds of fertilizer, one pound of fertilizer substitutes for 0.03 days of labour. Put on the basis of the equivalent of one ton of fertilizer, these values represent correspondingly the substitution of a ton of fertilizer for 9,184, 8,154, and 67 eight-hour days of total labour. The corresponding values derived from the square-root model are infinite, 3,9:4 and 1,149 eight-hour days Of these total labour days, 66 per cent account for human labour and the remaining for bullock labour.

Under quadratic model, with 20 pounds of fertilizer, 373.5 days of labour are required to produce 20 maunds of paddy. With 80 pounds of fertilizer 206.4 days of labour are required to produce the same output. The corresponding marginal rates of substitution for one pound of fertilizer are 3·79 and 1·47 days of labour, respectively. For square-root model, the corresponding substitution rates for one pound of fertilizer are 2·15 and 0 79 days of labour respectively.

For Chandukuri (Table 4) when $r=1$, under the quadratic model with 20 pounds of fertilizer, 48.1 days of labour are required to produce 15 maunds while 70.0 labour days are required to produce 20 maunds. The corresponding marginal rates of substitution are one pound of fertilizer for 0.84 days of labour in the former and 0.91 days of labour in the latter case. For the square-root model, the corresponding substitution rates are one pound of fertilizer for 0.46 days of labour and one pound of fertilizer for 0.57 days of labour, respectively.

As the input requirement of labour per acre is very high for Tirurkuppam, the substitution rates for labour are very high and appear less meaningful compared to those of Chandukuri.

Conclusions

While the data under the study are for experimental conditions and may somewhat overestimate the rate at which fertilizer substitutes for land and labour on all land, the marginal replacement rates are obviously high. As one can expect that individual farmer does not buy more fertilizer and use less land and less labour. He purchases the fertilizer and uses it on the given land area. In the aggregate sense and over time, however, fertilizer does become a substitute for these two resources, since the given output can be produced with fewer acres and less labor.

The high substitution rates for land also indicate the potentialities for increasing agricultural production in India where the acreage is severely restricted, through fertilizer application. With sufficient quantities of fertilizer available to farmers, the agricultural production can be significantly increased. The high substitution rates for labour could mean to indicate that small amounts of fertilizer can substitute for many days of labour. With abundant and cheap labour in present Indian agriculture, there may be little urgency in substituting fertilizer for labour. But, with plenty of fertilizer available in the future and its price being relatively cheaper to the cost of labour, one can reasonably wish to substitute fertilizer for labour.

Most important, however, we see that fertilizer has a high marginal rate of substitution for land. The rate declines as fertilizer level is higher, but it will certainly be large for sometime at the rate of use characterizing India. In a rough way, we could specify the extent to which investment should be in land reclamation or fertilizer facilities and distribution. The criteria is one of equating the fertilizer—land substitution rates with the relative costs of land development and fertilizer production. This analysis can be approached in a later paper.

The quadratic form appears to be preferable to square-root form for estimation purposes. At the currently low average level of fertilizer application in India, the square-root form estimates unrealistically high substitution rates. Generally, the substitution rates derived from the square-root form are higher than the corresponding ones derived from the quadratic model. But as the rate of fertilizer application increases, both forms give reasonably meaningful results.

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APPENDIX

The basic functions upon which our data are based are shown below where Y refers to maunds per acre of paddy while N and P refer, respectively, to pounds per acre nitrogen and phosphorus. (Wherever available, standard errors of the estimates are presented in brackets.) In estimating the optimum doses of N and P, the prices of N and P of were taken as Re. 0.7701 and Re. 0.6278 per pound, respectively, and the price of paddy as Rs. 10 per maund.

Location: *Tirurkuppam, Madras*

Quadratic Model ($R^2=0.9524$) :

$$Y=7.29476+\underset{(0.0067)}{0.08675N}+\underset{(0.0383)}{0.27540P}-\underset{(0.00002)}{0.00057N^2}-\underset{(0.00003)}{0.00377P^2}+\underset{(0.00016)}{0.00152NP}$$

Square-root Model ($R^2=0.9649$):

$$Y=8.65312+\underset{(0.2345)}{0.7645N^{\frac{1}{2}}}+\underset{(0.3448)}{1.17353P^{\frac{1}{2}}}+\underset{(0.0197)}{0.00420N}-\underset{(0.0425)}{0.13190P}+\underset{(0.0195)}{0.13932N^{\frac{1}{2}}P^{\frac{1}{2}}}$$

Location: *Chandukuri, Madhya Pradesh*

Quadratic Model ($R^2=0{\cdot}9475$).

$$Y=10.20425+0.08210N+0.38139P-0.00088N^2-0.00472P^2+0.00167NP$$

Square-root Model ($R^2=0.9892$):

$$Y=10.57172-0.00823N^{\frac{1}{2}}+2.07645P^{\frac{1}{2}}+0.01830N-0.17323P+0.10669N^{\frac{1}{2}}P^{\frac{1}{2}}$$

Optimum Doses of N and P

Location	*Quadratic model*			*Square-root model*		
	Optimum Dose		*Optimum Yield (Y) per acre*	*Optimum Dose*		*Optimum Yield (Y) per acre*
	Pounds of N per acre	*Pounds of P per acre*		*Pounds of N per acre*	*Pounds of P per acre*	
Tirurkuppam	63.27	41·02	19·41	26·86	23·71	15·27
Chandukuri	41·94	40·98	22·68	24·01	30 25	20·01

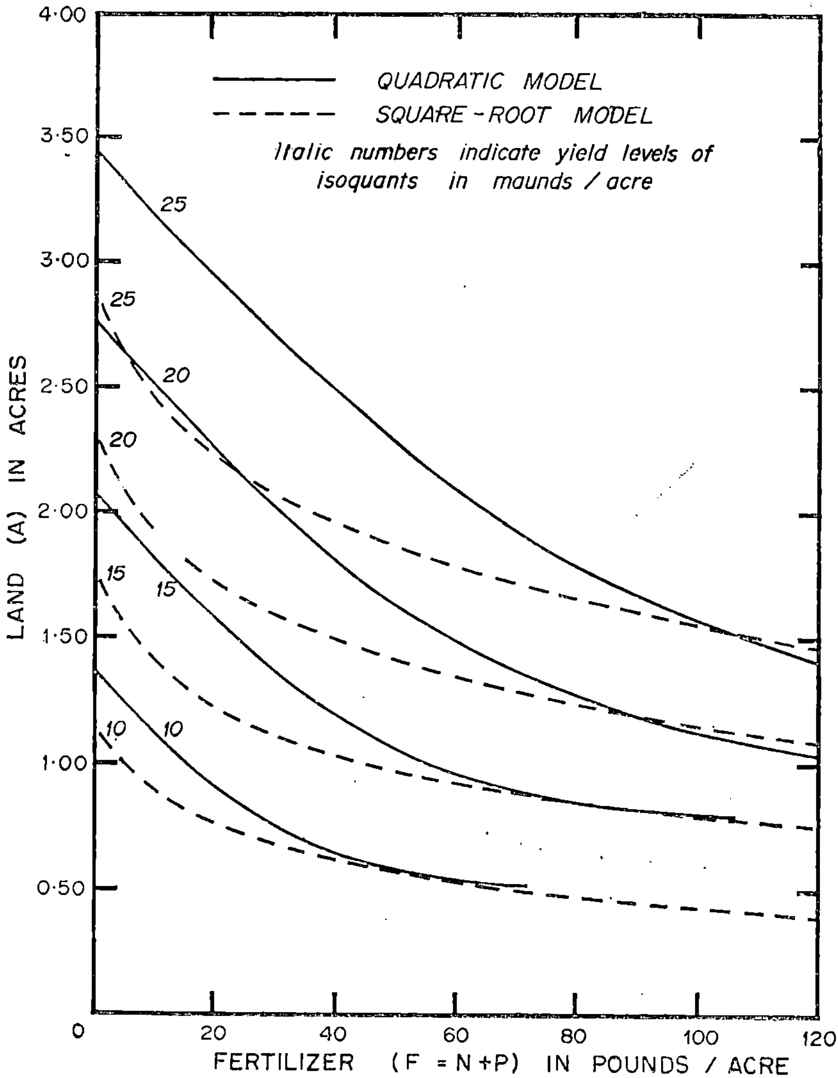

Fig. 1. Land-fertilizer isoquants for paddy when $N=P$ for Tirurkuppam.

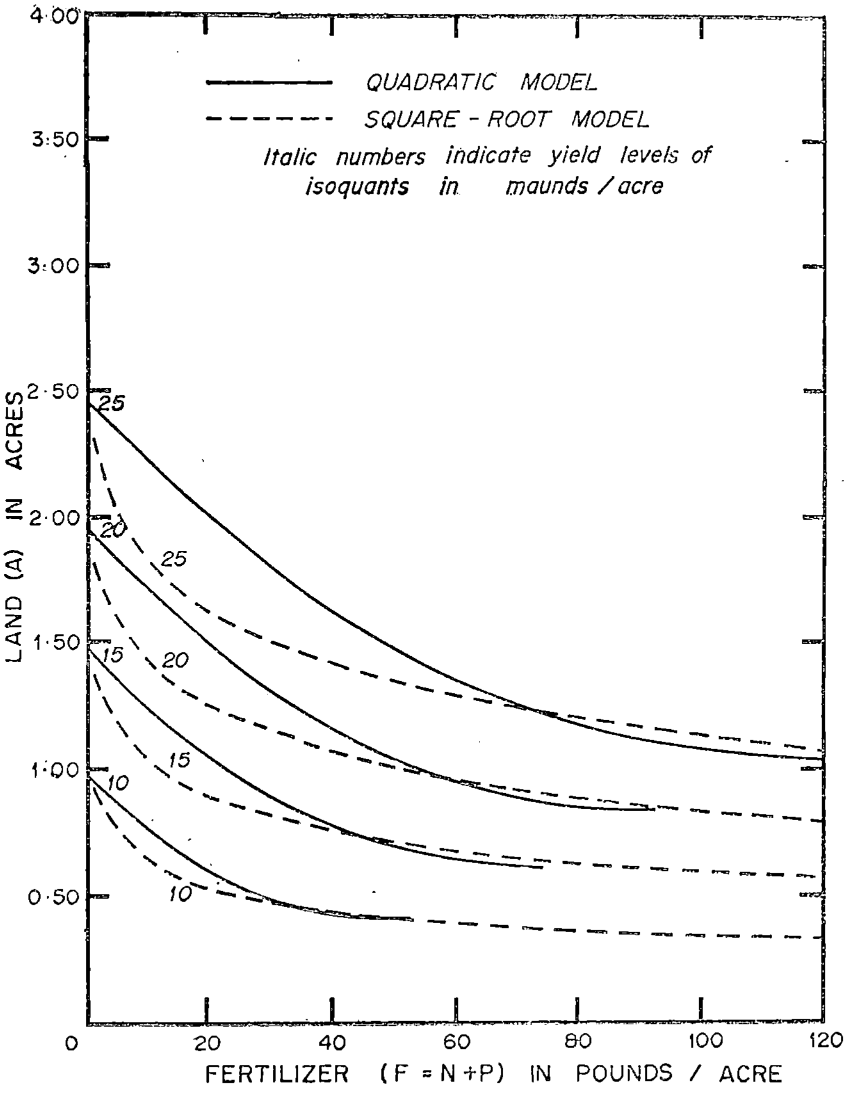

Fig. 2. Land-fertilizer isoquants for paddy when $N=P$ for Chandukuri.

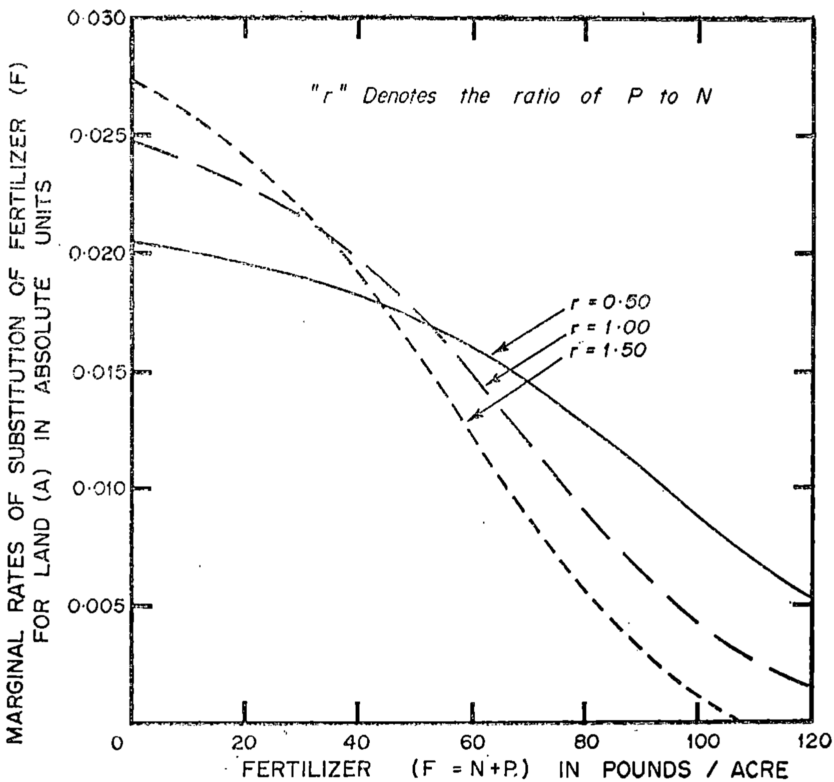

Fig. 3. Marginal rates of substitution of fertilizer (F) for land (A) for different values of r for the isoquant of 20 maunds per acre derived from quadratic model for Tirurkuppam.

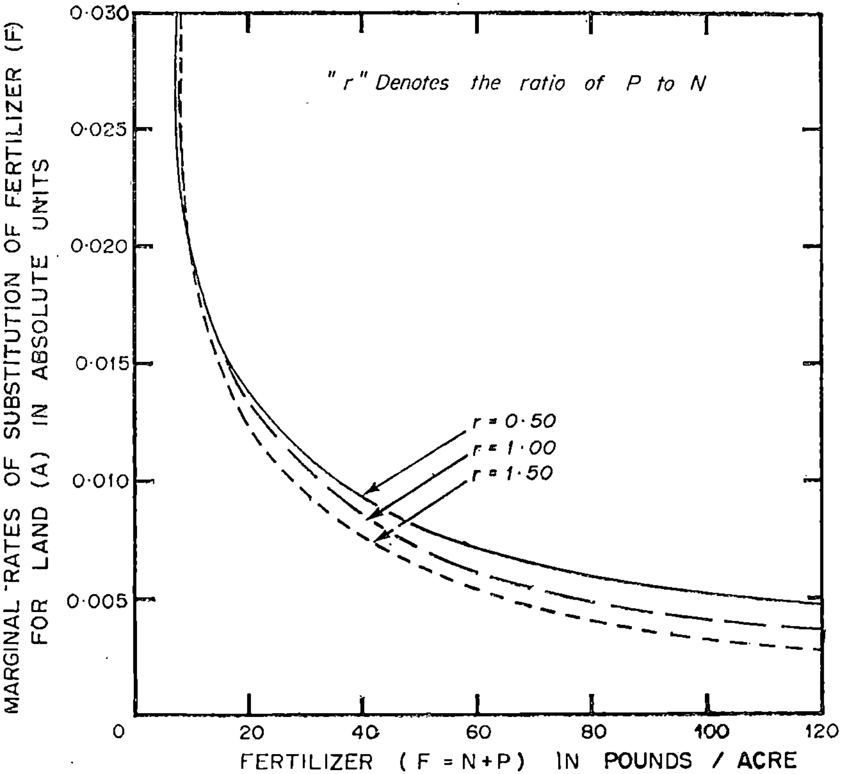

Fig. 4. Marginal rates of substitution of fertilizer (F) for land (A) for different values of r for the isoquant of 20 maunds per acre derived from square root model for Tirurkuppam.

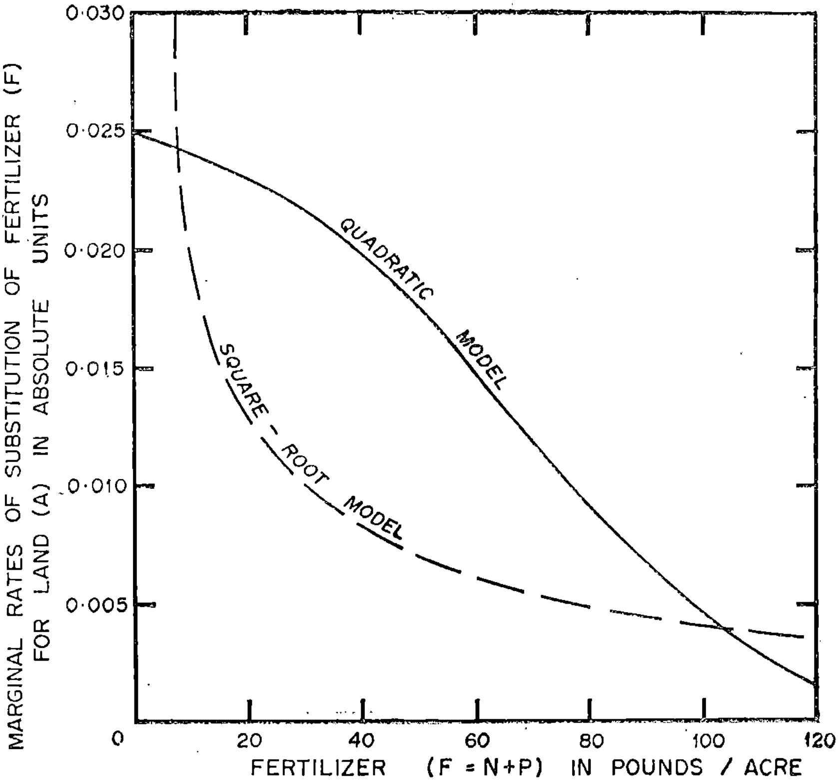

Fig. 5. Marginal rates of substitution of fertilizer (F) for land (A) for the isoquant of 20 maunds per acre derived from the two models when $N=P$ for Tirurkuppam.

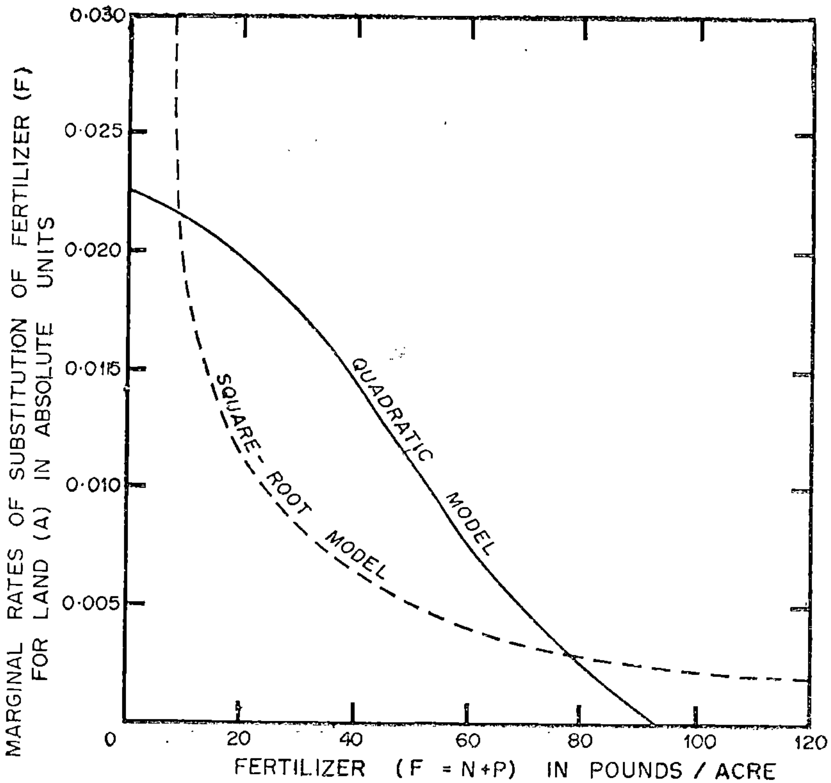

Fig. 6. Marginal rates of substitution of fertilizer (F) for land (A) for the isoquant of 20 maunds per acre derived from the two models when $N=P$ for Chandukuri.